Williamsville Central School District

Discipline: Mathematics
Course/Grade: **Pre-Calculus R** / High School
Final Exam: District Final Exam
Textbook: **Precalculus - Graphic, Numeric, Algebraic (6th Ed.)** By Demana, Waits, Foley and Kennedy
Pearson Education © 2004
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Course Description:

Topics of study in all pre-calculus courses include: fundamental concepts of algebra, solving equations and inequalities, functions and graphs, polynomial functions, rational functions and functions involving radicals, exponential and logarithmic functions, trigonometric functions, matrices, sequences and series, and conic sections.

Pre-Calculus R thoroughly combines algebra and geometry to prepare students to undertake the study of calculus. Since functions are the foundations of calculus, this course has been specifically developed to give the student a detailed understanding of elementary functions. The use of a graphing utility and the inclusion of realistic applications from the physical world, school environment, and from the quantitative world of mathematics, is an integral part of the fourth year mathematics course.

Pre-Calculus R is highly recommended preparation for motivated, average-to-very good students whose plans include the possibility of formal education beyond high school.

Required Prerequisite:

Successful completion of at least "Algebra 2 & Trigonometry R."

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Guide to Curriculum Related Vocabulary

Guaranteed and Viable Curriculum

Guaranteed: The guaranteed curriculum is what is **imperative** to teach – a curriculum that is communicated and assured to all groups; clear guidance to teachers regarding what knowledge is **expected** to be learned in courses or at grade levels.

Viable: a viable curriculum that can be realistically taught during the time available during the course of a school year. Its focus is on what is **essential vs. supplemental** to teach in a school year. It must be organized and sequenced to enable effective student learning – that is, to say, checking to make sure the essentials are being taught **AND** learned. The focus is on the standards that are most essential and demand the greatest amount of time.

Curriculum: the sequencing and pacing of essential declarative and procedural knowledge, common assessments along with the experiences students mush have with the content.

Power Performance Indicators

Power Performance Indicators: are essential parts of the curriculum and define the essential (inescapable) knowledge, understandings, skills, and processes of a particular course of study. They should be designated based on their endurance, leverage (capacity of the standard to be applied) and importance for higher level learning in the discipline.

Essential Components

Declarative Knowledge: Answers the questions, "What do students need to know and understand?" This includes: facts, concepts, principles, generalizations, cause/effect sequences, time sequences, and vocabulary terms.

Procedural Knowledge: Answers the questions, "What do students need to be able to do and at what level of application?" (i.e., Bloom's Taxonomy). This includes: skills and processes that result in construction of models, shaping of ideas, and internalization of knowledge (practice to achieve automaticity and fluency).

Key Vocabulary: Vocabulary deemed essential to the curriculum.

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Guide to Curriculum Design

Focus Questions
Focus questions provide specific content and facts about essential questions. They add depth and specificity, are answerable using the facts and materials of the unit, lead to particular understandings related to the topics of the unit, and provide for scaffolding leading to the essential questions.

Declarative Knowledge	Procedural Knowledge	Key Vocabulary
Answers the question… "What do students need to know and understand?" Includes facts, concepts, principles, generalizations, cause/effect sequences, time sequences, and vocabulary terms	Answers the question… "What do students need to be able to do and at what level of application?" Includes skills and processes that results in construction of models, shaping of ideas, and internalization of knowledge (Practice to achieve automaticity and fluency)	Vocabulary deemed essential to the curriculum

Williamsville's Learning Standard for Mathematics

In implementing the Pre-Calculus R curriculum, it is expected that students will identify and justify mathematical relationships, formally and informally. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the words *investigate, explore, discover, conjecture, reasoning, argument, justify, explain, proof,* and *apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

Investigate/Explore - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

Discover - Students will make note of possible patterns and generalizations that result from investigation/exploration.

Conjecture - Students will make an overall statement, thought to be true, about the new discovery.

Reasoning - Students will engage in a process that leads to knowing something to be true or false.

Argument - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

Justify/Explain - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

Proof - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems.

Apply - Students will use a theorem or concept to solve an algebraic or numerical problem.

Common Course Assessments

Assessment: is the means a teacher uses to determine whether or not students have learned the content, processes, and procedures required in the articulated guaranteed and viable curriculum. Assessment may be formative or summative in nature. It may also be used to screen or diagnose.

Formative Assessment: are designed to determine whether or not a student has grasped the curriculum that has been taught; it is assessment "for" learning and is administered at regular intervals; it is utilized to inform and adjust instruction "along the way." Formative assessments should be aligned to the summative assessment.

Benchmark assessments: are intended to measure the precise content of the curriculum that is intended to be learned in a given amount of time. They are typically administered about the time that grades are determined for a quarter or semester. Benchmark and common formative assessments are specific types of formative assessments. Examples: journal entries, exit tickets, performance tasks, quizzes, tests, projects

Common formative assessments are specifically designed by participating teachers of elementary grade level teams and secondary course/department teams who all teach the same content standards to their students. They provide a sharp focus for instruction and are directly linked to power standards.

Summative Assessment: occur at the end of a unit/course of study with the intent of evaluating student learning for reporting purposes. It is assessment "of" learning. Summative assessments are used to report final results to students, parents, and administrators. They typically support the assignment of grades and/or levels of proficiency. Examples: Unit tests, final examinations, final exhibitions

Screening Assessment: is an initial, first step to identify "red flags" and to inform whether a more thorough assessment is advisable. Example: Kindergarten screening test, ESL screening test

Diagnostic Assessment: is an in-depth assessment to identify special needs or areas where a student has a particular difficulty.

Rubric: A scoring guide that explains levels of performance and provides focus on the learning. A rubric should be designed to accompany all common assessments articulated in a curriculum. It serves as a guideline for rating student performance. Rubric types include holistic (general assessment of performance) and analytic (task specific).

Pre-Calculus R
Assessment Outline

Name of Assessment	Benchmark Formative	Common Formative	Summative	Screening	Diagnostic	Window of Admin.	Access of Results
Final Exam			X				

<u>**Unit 1:**</u> **Concepts of Algebra**

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 1	**Sect P-1** Sets of Numbers Interval Notation Properties of Exponents	Sets of Numbers – Complex, Imaginary, Real, Rational, Irrational, Integers, Whole, and Natural. Interval Notation (including compound intervals) Laws of Exponents – multiplying, dividing, power to a power, power of a product, power of a quotient, zero & negative	Identify all sets of numbers (including use of symbols) Interpret directions using sets of numbers Convert between interval and inequality notation Apply the laws of exponents to simplify expressions including expressions with multiple negative exponents	Complex numbers Imaginary numbers Real numbers Rational numbers Irrational numbers Integers Whole numbers Natural numbers Open Closed Bounded Unbounded Interval notation
Day 2	**Sect A-1** Radicals & Exponents	Properties of radicals Fractional exponents	Convert between radical and exponent form Simplify, add, subtract, multiply, and divide radical expressions of various indexes	Radicand Radical index *n*th root

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 3	**Sect A-2 & Sect 2-4** Polynomials	Addition, subtraction and multiplication of polynomials – including long division Division algorithm for polynomials	Simplify polynomials by adding, subtracting, multiplying, dividing. Include polynomials with multiple terms and degrees when adding and subtracting When multiplying, include monomial times monomial, monomial times binomial, binomial times binomial, binomial times trinomial, and trinomial times trinomial Divide polynomials using long division with divisor degree 1 or 2 (including those that result in a remainder other than 0)	Polynomial Leading coefficient Degree Standard form Quotient Remainder Dividend Divisor
Days 4–6	**Sect A-2** Factoring	Greatest common factor (GCF) Difference of two squares Trinomials Grouping techniques Sum and difference of two cubes Sum of two squares over complex #'s	Factor using monomial and binomial GCF Apply difference of two squares with monomial and binomial squares Factor trinomials with leading coefficient greater than or equal to 1. Trinomials could also be $x^2 - 7xy + 10y^2$ or $x^4 - 7x^2 + 10$ Apply various grouping techniques to factor polynomials with four or more terms Factor completely using multiple factoring methods Factor the sum of two squares over the set of complex numbers	Greatest Common Factor Difference of two squares Trinomials Grouping Sum and difference of two cubes Sum of two squares

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 7	**Sect A-3** Fractional Expression	Operations with rational expressions Compound rational expressions	Simply, add, subtract, multiply, & divide rational expressions

When adding and subtracting, use rational expressions that have both monomial and binomial denominators

For multiplying and dividing, have rational expressions that will simplify, so that the students are practicing factoring involving difference of two squares, sum and difference of two cubes, trinomials, and GFC

Simply complex fractions using the least common denominator (LCD). Include expressions with a binomial LCD that require multiple factoring to simplify | Rational expression Complex fraction LCD |

Notes: Include 2–3 days for review, quiz, & test.

<u>**Unit 2:**</u> **Solving Equations & Inequalities**

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Days 1–2	**Sect. P-5** Quadratic equations	Quadratic equations	Algebraically solve quadratic equations with a leading coefficient greater than or equal to one using the quadratic formula, factoring, completing the square and extracting the square root Graphically solve quadratic equations greater than or equal to one by using the calculator	Quadratic equation Roots Zeros Solutions *x*-Intercepts
Day 3	**Sect. P-3** Fractional equations Radical equations Absolute value eqs/ineq	Fractional equation Radical equations(Optional) Absolute value eqs/ineq(Optional)	Algebraically and graphically solve fractional equations. Use examples with extraneous roots. Algebraically and graphically solve radical equations. Use examples with extraneous roots.	Fractional equation Radical equation Extraneous roots
Day 4	**Sect. P-6** **Sect. 2-9** Quadratic inequalities Higher degree polynomial inequalities	Quadratic inequalities Higher degree polynomial inequalities that factor	Algebraically (using a sign test) solve a quadratic inequality and express solution in interval notation Use a calculator to graphically find interval solutions	

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 5	**Sect 2-9** Rational inequalities	Rational inequalities with variables in the denominator	Algebraically find interval solutions using a sign test Use calculator to graphically find interval solutions	
Day 6	**Sect. P-6** Applications of polynomial equations and inequalities	Set up an equation or inequality involving area and volume	Solve the applications algebraically or graphically including: constructing an open box, projectile motion, and enclosed rectangular area Applications should include equations or inequalities of degree two and three	

Notes: Include 2-3 days for review, quiz, and test.

Unit 3: Matrices (Chapters 10 & 12)

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 1	**Sect. 7-1 & 7-3** Systems of equations	Systems of equations	Solve systems of equations with Gaussian elimination methods. Include 2 equations and 2 unknowns, and 3 equations 3 unknowns by hand using Gaussian method and using using matricies RREF on the calculator	Systems of equations
Day 2	**Sect. 7-2** Matricies	Definition of matrix Matrix notation Matrix addition and subtraction Scalar multiplication	Determine the order of the matrix Addition and subtraction of two matrices by hand and calculator. Multiplying a matrix by a constant by hand and calculator.	Matrix Row Column Constant Element Entry Order
Day 3	**Sect. 7-2** Matrix multiplication	Matrix multiplication Identity Matrix Inverse Matrix	Multiply two matrices by hand and calculator of any order. Write an identity matrix for any order Verify 2 matrices are inverses by hand and calculator of any order.	Identity matrix Inverse matrix

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 4	**Sect. 7-2** Determinates	Definition of determinant Inverse matrix	Evaluate the determinant of a matrix 2x2 and 3x3 by hand. 2x2 or higher should be done on the calculator as well. Find the inverse matrix, 2x2 by hand. 2x2 or greater should be done on the calculator as well.	Determinant Square matrix Inverse matrix
Days 5-6	**Sect. 7-3** Row echelon form and reduced row echelon form	Row echelon form Reduced row echelon form	Apply row and reduced row echelon form to matrices. 3x4 should be as hard as it gets by hand in terms of row and reduced row echelon form. The calculator should be utilized as well for any order. Use matrices to solve systems of equations. Include word problems for 3 equations and 3 unknowns. In terms of analytic geometry, those questions are optional such as write the equation of a circle and parabola given 3 points, those are optional.	Row echelon form Reduced row echelon form

Notes: Include 2-3 days for review, quiz, and test.

Unit 4: Functions (Part I)

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Days 1–2	**Sect. 1-2** Functions and their properties	Definition of function Function notation Definition of relation Vertical line test Domain and range	Use the vertical line test to determine whether the formula defines y as a function of x Find the domain of a function algebraically, specifically focused on rational equations, radical equations, and fractional equations involving radicals Find the domain of a function graphically using the calculator Find the range of a function graphically using the calculator When finding domain and range, interval notation or set notation is acceptable. For example, $\mathbb{R}-\{4\}$ is just as acceptable as $(-\infty,4)\cup(4,\infty)$	Function Relation Vertical line test Domain Range
Day 3	**Sect. 1-2** Functions and their characteristics	Know the following characteristics Increasing Decreasing Constant Points of inflection Concavity Extrema	Looking at a graph, or given a function and using the calculator, find the local extrema, where the function is increasing, decreasing, and or constant, concavity and points of inflection	Increasing Decreasing Constant Points of inflection Concavity Extrema Local maximum Local minimum

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 4	**Sect. 1-2** Even and odd	Definition of even and odd functions Symmetry of even and odd functions	Graphically determine whether a function is even, odd or neither Plug in negative *x* to algebraically determine if a function is even, odd or neither	Symmetry Even functions Odd functions
Day 5	**Sect. 1-3** 12 basic functions	Know the 12 basic functions as listed: Identity, constant, square, square root, cubic, cube root, reciprocal, exponential, logarithmic, absolute value, greatest integer (floor), and signum	Graph the 12 basic functions by hand making sure that there are at least 5 points on the graph Know the characteristics of those functions, i.e. domain, range, increasing, decreasing, constant, extrema, *x*- and *y*-intercepts, asymptotes, and continuity Evaluate these functions (especially signum and greatest integer) at specific points	Identity Constant Square Square root Cubic Cube root Reciprocal, Exponential Logarithmic Absolute value Greatest integer (floor) Signum
Day 6	**Sect. 1-3** Piece-wise defined functions	Piece-wise defined functions	Graph piece-wise defined functions by hand and state characteristics as listed in day 5	Piece-wise defined function

Note: Include 2-3 days for review, quiz, and test

Unit 5: Functions (Part 2)

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 1	**Sect. 1-4** Building functions from functions	Algebra of functions: sum, difference, product and quotient of functions	Combine functions algebraically and determine their domain algebraically Evaluate functions, for example, $f(4)$, $f(x + h)$, $(f + g)(5)$, etc	
Day 2	**Sect. 1-4** Compositions of functions	Compositions of functions Decompositions of functions	Given functions $f(x)$ and $g(x)$, algebraically find $(f \circ g)x$ and its domain algebraically Given $h(x)$, find two functions f and g such that $h(x) = (f \circ g)x$	Composition
Days 3–4	**Sect. 1-4** Inverses of functions	Inverse relations Inverse functions and notation Horizontal line test One-to-one functions Domain and range of the inverse function – domain of the function is the range of the inverse, and the domain of the inverse is the range of the function	Given a function, find its inverse, $f^{-1}(x)$, by switching x and y and then solving for y.	Inverse relation Inverse function Horizontal line test One to one function Restricted domain

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
		Graphical representation of an inverse Two functions are inverses of each other if their compositions equal x Some functions have a inverse function only when you restrict the domain of the given function, hence the term, restricted domain	Given a graph, sketch the inverse relation by reflecting over the line $y = x$ Determine if the relation is one to one by using both horizontal and vertical line tests Prove two functions are inverses by showing $(f \circ f^{-1})x = (f^{-1} \circ f)x = x$	
Days 5–7	**Sect.1-5** Graphical Transformations	The relationship between changes in the equation of a functions and transformation in its graph including: vertical and horizontal shifts, reflections over the x- and y-axis, vertical stretches and shrinks, and absolute value of a function Note that horizontal stretch or shrink is optional but not required	Given a function, be able to state the basic function and list the transformations that are needed to produce that function. Students should be able to graph each transformation listed Note that when there is a reflection on the y-axis and a horizontal shift, you must shift first and then reflect on the y-axis Do not include functions such as $f(x) = \dfrac{x+3}{x-1}$ (this will be covered in Unit 7: Rational Functions) Include examples using vertex form of a parabola by completing the square such as $f(x) = 3x^2 + 12x + 11$	

Notes: Include 2-3 days for review, quiz, and test.

<u>Unit 6</u>: **Linear Functions**

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 1	**Sect. P-4** Cartesian Coordinate System	Slope Formula Equation of a line: 　　Slope – intercept form 　　Point – slope form 　　General Form (Ax + By + C = 0) 　　　(A, B, & C are integers, A > 0)	Find the slope of a line given two points (including points given in function notation) Write the equation of a line in various forms given two points (include point given in function notation) or one point and a slope Find slope of a line in general form using the formula $m = \dfrac{-A}{B}$ and y –intercept using the formula $y-\text{int} = \dfrac{-C}{B}$	Slope y-intercept General form
Day 2	**Supplement:** Working with Parallel & Perpendicular lines	Parallel lines have equal slopes Perpendicular lines have negative reciprocal slopes	Write the equation of a line parallel or perpendicular to a given line and through a given point in various forms	Parallel Perpendicular
Day 3	**Supplement:** Medians, Altitudes, & Perpendicular Bisectors	Midpoint Formula Equation of the line containing the median of a triangle from a given vertex	Find the midpoint of a segment using the midpoint formula Write the equation of the line that contains the altitude, median, or perpendicular bisector in various forms	Midpoint

Pacing	Section Topic	**Declarative Knowledge** *"What do students need to know and understand?"*	**Procedural Knowledge** *"What do students need to be able to do and at what level of application?"*	**Key Vocabulary**
		Equation of an Altitude of a triangle from a given vertex Equation of the line containing the perpendicular bisector of a triangle from a given vertex		
Day 4	**Supplement:** Distance Formulas	Distance between two points formula Distance from a point to a line formula.	Find the distance between two points using the formula Find the distance between a point and a line using the formula. Directed distance is optional Find the distance between two parallel lines	
Day 5	**Supplement:** Application Questions	How to find equations of various real – world applications that model linear functions	Write and solve linear equations given real-life applications including cost, revenue, profit, etc.	

Notes: Includes 2–3 days for review, quiz, & test.

Unit 7: Polynomial Functions

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 1	**Sect. 2-3** Polynomial Functions of higher degree	Definition of a polynomial function End behavior using limits Degree of a polynomial function Continuity of a polynomial function	Recognize polynomial functions Predict end behavior and general shape of a polynomial using degree and leading coefficient	Degree of a polynomial Leading coefficient Constant Standard form of a polynomial Leading term
Days 2–3	**Sect. 2-3 and Sect. 2-6** Finding Zeros by Factoring Multiplicity of a Polynomial	Zeros of polynomial Real zeros are *x*-intercepts Fundamental theorem of algebra Multiplicity of a zero and whether it crosses or is tangent to the *x*-axis Complex roots occur in conjugate pairs	Solve factorable polynomial functions of various degrees (including complex roots such as $f(x) = x^4 - 81$) Given zeros (rational and complex), write a polynomial function of least degree in standard form with real coefficients Given zeros and their multiplicity, write the equation or recognize the graph of the polynomial function	Rational number
Day 4	**Sect 2-4** Synthetic Division	Synthetic division (Horner's algorithm – optional)	Given polynomials *f(x)* and *d(x)* use synthetic division to find $\dfrac{f(x)}{d(x)}$	Synthetic division

Pacing	**Section** Topic	**Declarative Knowledge** *"What do students need to know and understand?"*	**Procedural Knowledge** *"What do students need to be able to do and at what level of application?"*	**Key Vocabulary**
			Answers should be expressed in the form $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$ (include polynomials with missing terms)	
Day 5	**Sect. 2-4** Remainder and Factor Theorems	Remainder theorem

Zeros of a polynomial

Factor theorem | Use the remainder theorem to find the remainder when $f(x)$ is divided by $x - k$

Use the factor theorem to determine whether a polynomial is a factor of another polynomial

Using the remainder, determine if the given number or is a zero of a polynomial function | Remainder theorem Factor theorem Zero |
| **Days 6–8** | **Sect 2-4, 2-5, and 2-6** | Possible rational zeros

Linear factorization

Complex zeros of a polynomial function | Find all complex zeros of a polynomial function using the possible rational zeros theorem, synthetic division, and quadratic formula. (Students may use the graphing calculator to find rational zeros from the list of possible rational zeros)

Write a polynomial function in factored form using linear factorization.

Functions should be degree 3 – 5. Also include zeros of multiplicity greater than 1 | |

Notes: Include 2–3 days for review, quiz, and test.

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Unit 8: Rational Functions

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Days 1–2	**Sect 2-7** Definition of a Rational function Limits Graphs of Rational Functions	Definition of a rational function Graphs of a rational function Transformations of the rational function Vertical asymptotes Horizontal asymptotes	Find the domain of a rational function Use limit notation to determine end behavior and behavior at vertical asymptotes Given a function, be able to state the basic function and list the transformations that are needed to produce that function Graph each transformation listed (include transformations covered in Unit 4) Do not include functions such as $f(x) = \dfrac{x+3}{x-1}$ (they will be covered in Unit 7: Rational Functions)	

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Days 3–5	**Sect. 2-7** Graphs of Rational Functions where numerator and/or denominator are degree ≥ 2.	Graphs with degree numerator less than or equal to the degree of the denominator Graphs with degree of the numerator greater than the degree of the denominator	Find the important characteristics of a rational function graph: ✓ x- and y-intercepts ✓ vertical asymptotes ✓ coordinates of any hole ✓ end behavior asymptotes ✓ domain and range ✓ increasing and/or decreasing extrema ✓ limits at vertical asymptotes ✓ limits at $\pm\infty$	

Notes: Include 2–3 days for review, quiz, and test

Unit 9: Exponential & Logarithmic Functions

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 1	**Sect. 3-1** Graph of the exponential function	Definition of an exponential function Graph of exponential function with base *e* (limit definition of *e* – optional) Notation of an exponential function Exponential function graph Domain and range	Find the domain and range of an exponential function Use limit notation to determine end behavior Given a function, be able to state the basic function and list the transformations that are needed to produce that function Graph each transformation listed. Should include transformations covered in unit 4 Determine if a function is increasing or decreasing Find horizontal asymptote Find *y*-intercept Write the equation of an exponential function given two points	Exponential function Base Exponent Initial value *e*

Pacing	**Section** Topic	**Declarative Knowledge** *"What do students need to know and understand?"*	**Procedural Knowledge** *"What do students need to be able to do and at what level of application?"*	**Key Vocabulary**
Day 2	**Sect. 3-1 & Supplement** Solving exponential equations and inequalities	Exponential equations and inequalities using change of base	Solve exponential equations and inequalities by getting bases equal (include equations that contain negative and fractional exponents; equations that require using the laws of exponents -- add, subtract, multiply exponents and equations that require exponents that are factorable quadratics)	
Day 3	**Sect. 3-3** Graph of the logarithmic function	Inverses of exponential functions Exponential and log Notation Definition of logarithmic functions Common logs Natural logs	Find the domain and range of an logarithmic function Use limit notation to determine end behavior Given a function, be able to state the basic function and list the transformations that are needed to produce that function Graph each transformation listed (include transformations covered in Unit 4) Determine if function is increasing or decreasing Find vertical asymptote Find x-intercept Apply inverse properties from unit 4 to exponential function to find log graph	Natural log Common log

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Days 4–5	**Sect. 3-3 and Sect. 3-4** Properties of Logs	Logarithmic expressions using properties of logs and natural logs Using inverses to simplify logs and exponential expressions Product rule, quotient rule, power rule change of base formula	Evaluate expression without a calculator that involve the properties of logs and/or inverses Use properties of logs to rewrite an expression as a sum or difference of logs or multiples of logs Use properties of logs to write expanded log expressions as a single log Use the change of base formula to write and evaluate log expression using base *e* or 10	
Days 6–7	**Sect. 3-5** Logarithmic & exponential equations and inequalities	Logarithmic equations & inequalities Exponential equations & inequalities	Solve algebraically logarithmic equations and inequalities, including those that use properties of logs, have extraneous roots, and/or require factoring Problems should include both calculator and non-calculator questions	
Days 8–9	**Sect. 3-6** Application Problems	Compounded interest Monthly payments Total interest paid	Solve application questions using properties of logs and/or exponents as they relate to interest and other finance problems	Compound interest Continuous interest

Notes: Include 2–3 days for review, quiz, and test

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Unit 10: Trigonometric Functions

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Days 1–2	**Sect. 4-2** and **Sect. 4-3** Angles and their Measure Unit Circle Six Trig Functions	Degrees and radians Exact value of a trigonometric function sine, cosine, and tangent graphs	Convert between degrees and radians Find the exact value of a trigonometric functions including those in radians without use of calculator Graph one period of sin, cos, and tan graphs	Degree Radian Sine Cosine Tangent Cosecant Secant Cotangent
Days 3–4	**Sect. 4-5** and **Sect. 4-7** Reciprocal Functions & Inverse Functions	Graphs of sec, csc, and cot Domain & range of reciprocal functions Graphs of $y = \sin^{-1}(x), y = \cos^{-1}(x),$ *and* $y = \tan^{-1}(x)$ Domain & range of inverse trig functions	Find the domain and range of the reciprocal trig functions and inverse trig functions Graph one cycle of reciprocal trig graphs Find exact value of inverse trig functions at a value (positive and negative)	Period Amplitude Phase Shift Exact Value

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 5	**Sect. 4-4 and Sect. 4-5** Transformations of trigonometric graphs	Amplitude Frequency & Period Phase shifts & vertical shifts Reflections over the x-axis	Given a function, be able to state the basic function and list the transformations that are needed to produce that function Graph each transformation listed Graph one cycle of any of the 6 functions with transformations especially with frequency and phase shift changes	

Notes: Include 2–3 days for review, test, quiz.

Unit 11: Polar Coordinates (Optional)

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 1	**Section 6-4** Polar Coordinates	Polar coordinates Convert between rectangular and polar coordinates and equations	Plot points using polar coordinates Convert between rectangular and polar coordinates and equations	Polar Axis Pole Rectangular Coordinate Polar Coordinate
Day 2	**Section 6-5** Polar equations and their Graphs	Polar equations by plotting points and by using a graphing calculator	Graph polar equations by plotting points and by using a graphing calculator	Polar equation Rose Curve Limacon Cardioid Spiral
Days 3–4	**Section 6-6** Complex Plane and DeMoivre's Theorem	Polar and rectangular forms of a complex number CIS notation Plot points in the complex plane Products and quotients of complex numbers in polar form DeMoivre's theorem	Convert between polar and rectangular form of a complex number Write $r(\cos\theta + i\sin\theta)$ using rCIS notation Plot points in the complex plane and determine r and θ Determine the products and quotients of complex numbers in polar form and be able to convert to rectangular form Use DeMoivre's Theorem to raise complex numbers to powers	Complex number in rectangular form Complex number in trigonometric form DeMoivre's theorem

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Days 5–6	**Section 6-6** (continued)	Complex roots	Determine the complex roots of a complex number using the formula	Complex root

Note: Include 2-3 days for review, quiz and test.

Unit 12: Series and Sequences and Math Induction(All Series and Sequences are Optional, but Induction is Mandatory)

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary

Days 1- 2	**Sect. 9-4 and Supplement**
	Arithmetic Series and Sequences
		Definition of :
		    Infinite sequences
		    Finite sequences
		    Arithmetic sequences
		    Arithmetic series
		
		Sigma notation
		
		Explicit formula
		
		Recursive formula
		
		Common difference
			Identify an arithmetic sequence
			
			Determine the common difference in an arithmetic sequence
			
			Find the formula for the nth term of an arithmetic sequence
			
			Find the nth term given a recursive formula
			
			Determine a specified term of an arithmetic sequence
			
			Determine the sum of the first *n* terms of an arithmetic series
			
			Represent the sum of an arithmetic series using sigma notation
				Infinite sequence
				Finite sequences
				Arithmetic sequence
				Arithmetic series
				Common difference
				Explicit formula
				Recursive formula
				n^{th} term
				Sigma

Days 3-4	**Sect. 9-4 and Supplement** Geometric Series and Sequences	Geometric sequences Geometric series Common ratio	Identify a geometric sequence Determine the common ratio in a geometric sequence	Geometric sequences Geometric series Common ratio Converge Diverge

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
		Behavior of a series with an infinite number of terms

Limits of infinite series | Find the formula for its nth term

Determine a specified term of a geometric sequence

Determine the sum of the first n terms of a geometric series

Represent the sum of a geometric series, using sigma notation

Determine if the behavior of a series increase without limit, decreases without limit, oscillates, or approach a limit

Determine the limit of a geometric series when $\lvert r \rvert < 1$ | |
| **Days 5-6** | **Sect. 9-4 and Supplement** Math Induction Sum Proofs | Principle of mathematical induction

Using mathematical induction to prove validity of a series | Use mathematical deduction to prove that the given arithmetic statement is true (*i.e,* show the anchor is true, provide the inductive hypothesis, show the inductive steps, and writing a conclusion) | Mathematical induction Anchor Induction hypothesis Induction step Sum |
| **Days 7-8** | **Sect. 9-4 and supplement** Math Induction Divisibility proofs | Principle of mathematical induction

Using mathematical induction to prove divisibility of a polynomial and an exponential | Use mathematical deduction to prove that the given divisibility statement is true (*i.e.*, show the anchor is true, provide the inductive hypothesis, show the inductive steps, and writing a conclusion) | Divisibility |

Notes: Include 3 days for review and test.

<u>Unit 13</u>: Circles and Parabolas

Pacing	Section Topic	Declarative Knowledge "What do students need to know and understand?"	Procedural Knowledge "What do students need to be able to do and at what level of application?"	Key Vocabulary
Day 1	**Supplement** Standard form of a circle	Conic sections Definition of a circle Standard form of the equation of a circle Conic sections are formed by the intersection of a plane with a right circular cone (of two nappes) Degenerate conic sections can be obtained from cross sections of a degenerate cone Locus definition of a circle	Graph the equation of the circle given in standard form Write the equation of the circle in standard form given certain conditions (try to use the following: concentric with, endpoints of the diameter, tangent to a line, endpoints of a chord, center is the intersection of two lines, etc.)	Right circular cone (of two nappes) Conic sections Plane Degenerate conic sections Radius Center Standard form of a circle Locus definition Concentric Tangent Chord Diameter
Day 2	**Supplement** General form of a circle	General quadratic equation in x and y General form of the equation of a circle All quadratic equations in x and y can be written in the general form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where all the coefficients are constants and at least one of the first three is different from zero	Complete the square in two variables to go from general form to standard form Expand and multiply the equation of a circle in standard form to produce the equation of that circle in general form	General form of a quadratic equation in x and y General form of the equation of a circle Null circle

Pacing	**Section** Topic	**Declarative Knowledge** *"What do students need to know and understand?"*	**Procedural Knowledge** *"What do students need to be able to do and at what level of application?"*	**Key Vocabulary**
		General equation of a circle has $A = C = 1$, and $B = 0$ thus $x^2 + y^2 + Dx + Ey + F = 0$	Consider problems such that after completion of the square, if the right side is less than zero we have no graph, if the right side is zero we have a null circle, and if the right side is positive that is a circle	
Day 3	**Supplement** Advanced circle problems	Standard form of the equation of a circle	Write the standard form of the equation of a circle given more complicated conditions such as: center is on a specific point and the circle passes through another point, passing through three points, center is on a given line and the circle is tangent to both *x*- and *y*- axes, center is on a given line and circle passes through two points, etc.	
Day 4	**Sect. 8-1** Geometry of a parabola	Definition of a parabola Standard form of a parabola Locus definition of a parabola Rotate the parabola 90 degrees, 180 degrees, and 270 degrees to derive all four equations	Graph a parabola in standard form with center at (h,k) by translating the parabola from center $(0,0)$	Parabola Directrix Focus Vertex Focal width Axis of symmetry Focal length
Day 5	**Sect. 8-1** Standard form of the equation of the parabola	Standard form of a parabola	Write the standard form of a parabola that satisfies certain given conditions (include a variety of problems that have the following combinations of: coordinate of the focus, equation of the directrix, which way it is opening, coordinate of the vertex, equation of the axis of symmetry)	

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 6	**Sect. 8-1** General form of the equation of the parabola	General form of the equation of the parabola	Complete the square of a parabola in general form to obtain the standard form Expand the equation of the parabola in standard form to obtain the general form After completing the square, talk about all scenarios: if only one x or y term is present then you will have either two horizontal or two vertical lines or no graph if the right side is negative, and if both x and y variables are present you have either a parabola or no graph if the right side is negative	

Notes: Include 2–3 days for review, quiz and test.

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<u>**Unit 14:**</u> **Conics – Ellipses and Hyperbolas**

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Days 1-2	**Sect. 8-2** Ellipses Finding the components of an ellipse and graphing the ellipse	Definition of ellipse Standard form of the equation $(a^2 > b^2)$: $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ $\ or\ \dfrac{(y-k)^2}{a^2}+\dfrac{(x-h)^2}{b^2}=1$ Major axis = 2a Minor axis = 2b Vertices $(h\pm a,k)\ or\ (h,k\pm a)$ Foci $(h\pm c,k)\ or\ (h,k\pm c)$ Latus rectum $=\dfrac{b^2}{2a}$ Eccentricity $=\dfrac{c}{a}$ Relationship between a, b and c : $a^2=b^2+c^2$ or $c^2=a^2-b^2$ General form	Given the equation of an ellipse in standard form (with center at any (h, k)), find the coordinates of the vertices, the coordinates of the foci, the coordinates of the endpoints of minor axis, the length of the latus rectum, the eccentricity, and make an accurate graph of the ellipse Complete the square for any ellipse equation provided in general form to obtain the standard form of the ellipse equation	Definition of ellipse Standard form General form Major axis Minor axis Vertices Foci Latus rectum Eccentricity Center
Day 3	**Sect. 8-2** Ellipses Finding the equation of an ellipse given certain information		Given data about the ellipse, determine the equation of the ellipse in standard form and/or general form	

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Days 4-5	**Sect 8-3** Hyperbola Finding the components of a hyperbola and graphing the hyperbola	Definition of hyperbola Standard form of the equation: $$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \ \ or \ \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$ Transverse axis = 2a Conjugate axis =2 b Vertices $(h\pm a,k) \ or \ (h,k\pm a)$ Foci $(h\pm c,k) \ or \ (h,k\pm c)$ Latus rectum $=\dfrac{b^2}{2a}$ Eccentricity $=\dfrac{c}{a}$ Relationship between a, b and c : $c^2=a^2+b^2$ Asymptotes of hyperbola: $$(y-k)=\pm\frac{b}{a}(x-h) \ or \ (y-k)=\pm\frac{a}{b}(x-h)$$ General Form	Given the equation of a hyperbola in standard form (with center at any (h, k)), find the coordinates of the vertices, coordinates of the foci, coordinates of the endpoints of conjugate axis, the length of the latus rectum, the eccentricity, the equations of the asymptotes, and make an accurate graph of the hyperbola using the fundamental rectangle Complete the square for any hyperbola equation provided in general form to obtain the standard form of the hyperbola equation	Definition of hyperbola Standard form General form Transverse axis Conjugate axis Vertices Foci Latus rectum Eccentricity Center Asymptotes Fundamental rectangle
Day 6	**Sect. 8-3** Hyperbola Finding the equation of a hyperbola given certain information		Given data about the hyperbola, determine the equation of the hyperbola in standard form and/or general form	

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 7	**Supplement** The Degenerate case of the ellipse and hyperbola	A quadratic equation in general form: $Ax^2 + By^2 + Cx + Dy + F = 0$ which has no xy term and whose x^2 coefficient and y^2 coefficient have the same sign and $A \neq B$ represents an ellipse in standard form a point or no graph • If after completing the square, the right is >0, then you get an ellipse • If after completing the square, the right is = 0, then you get a point • If after completing the square, the right is < 0, then you get no graph A quadratic equation that has no xy term and whose x^2 and y^2 coefficients have the opposite signs represents a hyperbola in standard position or 2 intersecting lines To determine if you have a hyperbola or 2 intersecting lines, complete the squares and look at the number on the right of the equation: • If the right number is either positive or negative , the equation is a hyperbola • If the right number is zero, the equation represents 1 or 2 lines	Given an equation determine by completing the square, determine if the equation represents a hyperbola, ellipse, a point, no graph, a line or 2 intersecting lines Identify the resulting equations or points	Degenerate Case

Notes: Include 3 days for review, quiz and test.

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Unit 15: Limits

Pacing	Section Topic	**Declarative Knowledge** *"What do students need to know and understand?"*	**Procedural Knowledge** *"What do students need to be able to do and at what level of application?"*	**Key Vocabulary**
Day 1	**Sect. 10-3** Definition of a Limit	Definition of a limit Finding limits of continuous functions Types of discontinuity - graphically	Use definition of limit to find limits given a graph (graphs should include continuous and non-continuous functions) Looking at a graph find $\lim\limits_{x \to c^-} f(x)$, $\lim\limits_{x \to c^+} f(x)$, $\lim\limits_{x \to c} f(x)$, and $f(c)$ Find limits algebraically of continuous functions such as linear, polynomial, rational, trig, and exponential	Continuous function Left-hand limit $\lim\limits_{x \to c^-} f(x)$ Right-hand limit $\lim\limits_{x \to c^+} f(x)$
Day 2	**Sect. 10-3** Limits of Non-Continuous Functions Properties of Limits	Algebra techniques for finding limits Limit of a: • sum • difference • product • quotient • power or root	Find limits of non-continuous functions using algebra techniques including factor and cancel, rationalizing, and properties of limits Find limits of piece-wise functions graphically and algebraically	
Day 3	**Sect. 10-3** Limits at Infinity	End behavior of a function Limits at positive and negative infinity	Find limits at positive and negative infinity using the end behavior model Functions should include: linear, polynomial, rational, and exponential	

Notes: Include 2–3 days for review and limits.

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Unit 16: **Derivatives(Anything in terms of the Second Derivative and Concavity are Optional)**

Pacing	Section Topic	Declarative Knowledge *"What do students need to know and understand?"*	Procedural Knowledge *"What do students need to be able to do and at what level of application?"*	Key Vocabulary
Day 1	**Sect. 10-1** The Derivative	Definition of the derivative using limits Definition of the difference quotient Derivative notations Derivatives of polynomial functions	Using the definition of the derivative, write the derivative of a polynomial function Using the definition of the difference quotient to write the difference quotient of a polynomial function	Derivative $\dfrac{dy}{dx} = f'(x) = y'$ Average rate of change Instantaneous rate of change Difference quotient
Day 2	**Supplement** Finding Derivatives using Rule of Derivatives	Short cut method for derivatives of a: • constant • power • sum or difference Second derivative	Using the short cut method to find the first and second derivatives of a function Evaluate first derivative to find rate of change at a point on the curve	
Day 3	**Supplement** Writing the Equation of a Tangent line	Slope of a line tangent to a curve Tangent line to a curve	Use the derivative to find the slope of the line tangent to a curve Write the equation of a line tangent to a curve at specified point	Tangent

Pacing	Section Topic	**Declarative Knowledge** *"What do students need to know and understand?"*	**Procedural Knowledge** *"What do students need to be able to do and at what level of application?"*	**Key Vocabulary**
Day 4-5	**Supplement** Curve Sketching	Slope of a line tangent to a curve Increasing and decreasing functions and their derivatives Concavity and the second derivative Points of inflection(Optional)	Use characteristics about the first derivative to sketch functions. Use first derivative test to find possible local extrema and intervals where function is increasing and decreasing Use sign chart to find points of inflections and intervals where function is concave up and concave down(Optional) Use information about the first and second derivatives to sketch functions , without knowing the equation of the function, (Optional)	Increase Decrease Maximum Minimum Inflection point Concavity
Day 6	**Supplement** Application of the derivative	Optimization Applications – area, volume, projectile motion, Velocity Acceleration(Optional)	Find the max and min of a function using the derivative Find instantaneous velocity Find acceleration of a function(Optional)	Optimization Minimum Maximum Instantaneous rate of change

Notes: Include 2–3 days for review, quiz, and test.